

# A Corrugated Waveguide Phase Shifter and its Use in HPM Dual-Reflector Antenna Arrays

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**Abstract**—An analysis, with computations and experimental verification, is given which shows that an axially compressible or extendable thin-wall corrugated metallic cylinder can be used as a phase shifter to achieve  $\pm 180^\circ$  phase control. Employment of such a device in an array of HPM dual-reflector antennas to achieve maximum in-phase array radiation is then briefly outlined.

## I. INTRODUCTION

THE NEED for and type of phase-shifters to control the phase of the field emitted by a single HPM (High Power Microwave) antenna, so as to realize maximum (in-phase) addition of the sum of the fields emitted by an array of such antennas, has been concisely reviewed with the conclusion that a phase-shifter in the form of an adjustable-length transmission line having no moving-contacts be studied [1]. Such a phase-shifter, in the specific form of thin-wall metallic corrugated cylindrical tube that renders it axially compressible or extendable, was therefore investigated both theoretically and experimentally with the results reported herein.

## II. ANALYSIS

Consider then the corrugated waveguide phase shifter (denoted henceforth as a CWPS) depicted in Fig. 1(a) showing the actual cylinder and its mathematical-idealized model, and in Fig. 1(b) showing the extremes of its compression and expansion. Since the cylinder has many corrugations per free space wavelength, the electromagnetic wave propagation it can support can be determined using the well known surface impedance model [2]. Such a model shows that the fields are hybrid in nature (i.e., TE and TM components exist) and, for the case of the cylinder being connected to a metallic-waveguide of the same inner diameter,  $2a$ , and carrying only the dominant  $TE_{11}$  mode that is polarized along the  $x(\phi = 0)$  direction, this hybrid mode is the  $HE_{11}$  whose field components can be written concisely as [3], [4]:

$$\begin{pmatrix} H_Z \\ E_Z \\ E_\rho \\ E_\phi \\ H_\rho \\ H_\phi \end{pmatrix} = \begin{pmatrix} H_1 J_1(X) \cos \phi \\ H_1 Z_{TE} R_S J_1(X) \cos \phi \\ \varepsilon_0 [J_1(X)/X + R_S J'_1(X)] \cos \phi \\ -\varepsilon_0 [R_S J_1(X)/X + J'_1(X)] \sin \phi \\ (\varepsilon_0/Z_{TE}) [R_S J_1(X)/X + (\lambda_g/\lambda_0)^2 J'_1(X)] \sin \phi \\ (\varepsilon_0/Z_{TE}) [R_S (\lambda_g/\lambda_0)^2 J'_1(X) + J_1(X)/X] \cos \phi \end{pmatrix} \quad (1)$$

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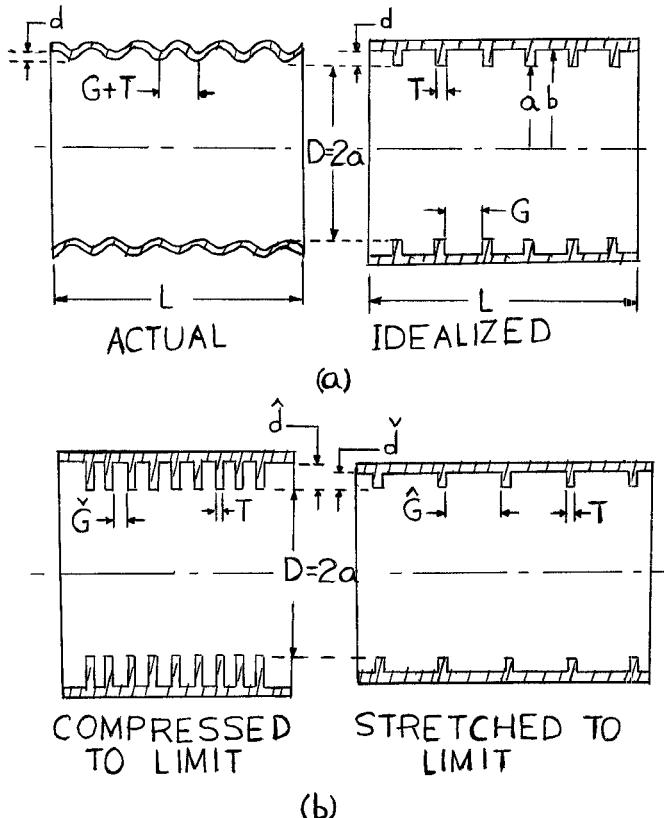


Fig. 1. Geometry of CWPS (corrugated waveguide phase-shifter). (a) Actual and mathematical equivalent. (b) Case 1: compressed to limit. Case 2: stretched to limit.

for  $0 \leq \rho \leq a$  and:

$$\begin{pmatrix} E_Z \\ H_\rho \\ H_\phi \end{pmatrix} = -[H_1 Z_{TE} R_S J_1(E)]/M \begin{pmatrix} N \cos \phi \\ [(jN)/(Z_0 C \xi)] \sin \phi \\ [(jQ)/Z_0] \cos \phi \end{pmatrix} \quad (2)$$

for  $a \leq \rho \leq b$ , where:  $X = E(\rho/a)$ ,  $\varepsilon_0 = (-jZ_0 H_1 C)/E$ ,  $\xi = \rho/a$ ,  $Z_0 = 120\pi$  ohms,  $Z_{TE} = Z_0(\lambda_g/\lambda_0)$ ,  $N = [J_1(C\xi) Y_1(CW) - Y_1(C\xi) J_1(CW)]$ ,  $Q = [J'_1(C\xi) Y_1(CW) - Y'_1(C\xi) J_1(CW)]$ ,  $M = [J_1(C) Y_1(CW) - Y_1(C) J_1(CW)]$  with  $J_1(X)$  and  $Y_1(X)$  being Bessel functions of order one of the first and second kind, respectively, with argument  $X$ , and  $J'_1(X)$  and  $Y'_1(X)$  indicate their derivatives with respect to  $X$ . In (1) and (2), the arguments  $(\rho, \phi, Z)$  of all the field components are understood,  $\exp[j(\omega t - \beta Z)]$  propagation is assumed, with  $\beta = 2\pi/\lambda_g = \omega/v_P$  being the propagation factor,  $\lambda_g$  the guide wavelength and,  $v_P$  the phase velocity

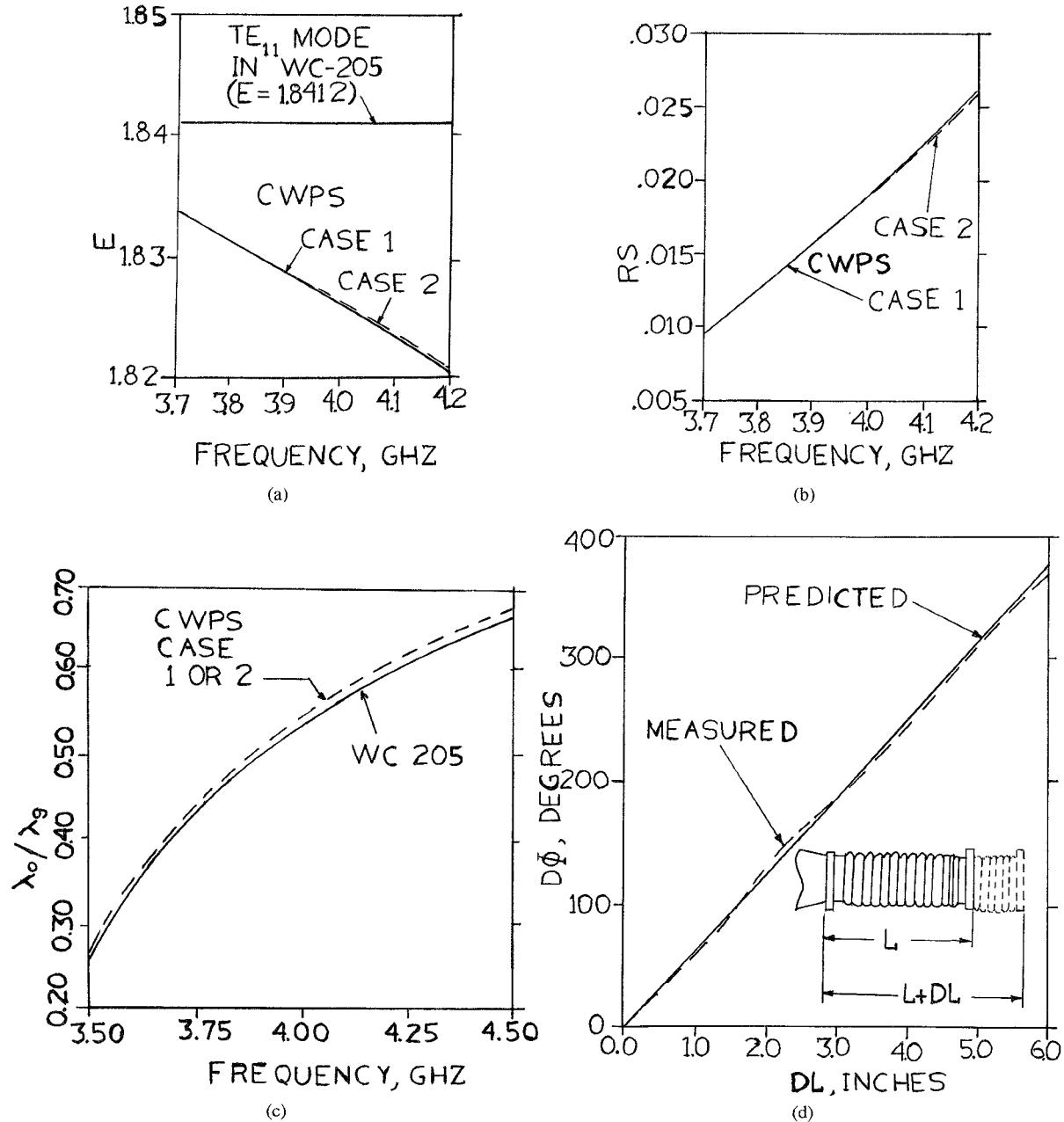


Fig. 2. (a) Computed eigenvalue,  $E$ , and (b) spherical hybridicity factor,  $R_S$ , of subject CWPS ( $D = 2.05''$ ,  $T = 1/32''$ ; Case 1:  $G = \bar{G} = 1/32''$ ,  $d = \bar{d} = 3/32''$ ; Case 2:  $G = \bar{G} = 3/32''$ ,  $d = \bar{d} = 2/32''$ ) over 4-GHz band. (c) Computed guide wavelength of subject CWPS and comparison with that of WC-205. (d) Predicted and measured phase delay of subject CWPS at 3.95 GHz.

where:

$$\beta/k = v_P/c = \lambda_0/\lambda_g = [1 - (E/C)^2]^{1/2} \quad (3)$$

where  $C = (2\pi a)/\lambda_0$ , and with  $\lambda_0$  and  $c$  being the wavelength and speed of light in free space, respectively, and  $E$  is the to-be-determined eigen-value. To obtain the value of  $E$ , one must solve the characteristic equation of:

$$\begin{aligned} \frac{C J'_1(E)}{E J_1(E)} - \frac{(C^2 - E^2)}{CE^3} \frac{J_1(E)}{J'_1(E)} \\ = [1 + (T/G)] \frac{[J'_1(C)Y_1(CW) - Y'_1(C)J_1(CW)]}{[J_1(C)Y_1(CW) - Y_1(C)J_1(CW)]} \end{aligned} \quad (4)$$

which follows by making the tangential field components be continuous at  $\rho = a$  (i.e., equating  $E_Z$ ,  $H_Z$ ,  $E_\phi$  and  $H_\phi$  of (1) to their corresponding fields of (2) at  $\rho = b$ ). In (4),  $W = b/a = 1 + DW$  (where  $DW = d/a$ ) and  $R_S$  is the spherical-hybridicity factor (defined as the ratio of peak axial-electric to peak axial-magnetic fields on-axis divided by  $Z_{TE11}$ ):  $R_S = E_Z(0, 0, Z)/[Z_{TE11}H_Z(0, \pi/2, Z)]$ , where  $Z_{TE11}$  is the characteristic wave impedance of the TE<sub>11</sub> mode  $Z_{TE11} = (Z_0 \lambda_{gTE11})/(\lambda_0)$ , with  $\lambda_{gTE11} = \lambda_0/[1 - (E_0/C)^2]^{1/2}$ , and  $E_0 = 1.8412$ , and where  $R_S$  is determined from:

$$R_S = -[E J'_1(E)]/J_1(E) \quad (5)$$

Hence, one need only solve (4) for  $E$  and then use (5) to obtain  $R_S$ . These being known, all the field components are then determined via (1) and (2). If the corrugation depth is vanishingly small ( $d = 0$ , i.e.,  $W = 1$ ) then one obtains from (3):  $J'_1(E) = 0$ , so  $E = E_0 = 1.8412$  and hence from (4)  $R_S = 0$ , which is the result for the smooth wall guide, as it should be. Here, the corrugation depth is small but not zero, so one would expect  $E$  to be close to  $E_0$  and  $R_S$  close to zero as will now be shown, both analytically and then via computer. Thus, a quick analytical solution for  $E$  can first be obtained for the shallow corrugation case by using the Taylor series-type approximations in (4) since  $E = E_0 + DE$  and  $W = b/a = (a + d)/a = 1 + DW$ , with  $DW = d/a \ll 1$ , so, for example,  $Y_1(CW) = Y_1(C) + C(DW)Y'_1(C)$ , etc., which, with Bessel function identities, gives

$$DE = -DW[(C/E_0)^2 - 1]/\{E_0[1 + (T/G)](1 - E_0^{-2})\} \quad (6)$$

### III. COMPUTATIONS

To demonstrate the feasibility of the CWPS a model having  $2a = 2.05''$  (corresponding to a smooth wall WC-205 guide, where WC denotes waveguide-circular) with  $T = 1/32''$  and  $\check{G} \leq G \leq \hat{G}$ , with  $\check{G} = 1/32''$ ,  $\hat{G} = 3/32''$ , and  $\check{d} \leq d \leq \hat{d}$ , with  $\check{d} = 2/32''$  and  $\hat{d} = 3/32''$  was employed, where the extreme-compressed condition ( $G = \check{G}$ ,  $d = \check{d}$ ) and the extreme-expanded condition ( $G = \hat{G}$ ,  $d = \hat{d}$ ) are referred to as Cases 1 and 2, respectively. Note that  $2d + G$  is constant at  $7/32''$ . The corresponding values of  $T/G$  and  $DW = d/a$  are then 1.00 and 0.09146 for Case 1, and 0.3333 and 0.06098 for Case 2. So, at  $f = 3.950$  GHZ ( $\lambda_0 = 2.990''$ ,  $C = 2.1538$ ) (6) gives  $DE = (-0.2839 DW)/[1 + (T/G)]$ , and  $DE \approx -0.0130$  for both Cases 1 and 2. The corresponding values of  $E$  are then both equal to  $1.8412 - 0.0130 = 1.828$ . Next, a computer solution to (4) was obtained over the 3.400 to 7.500 GHZ range and gives the results of Fig. 2a over the 3.700 to 4.20 GHZ band, and these, with (5) give the corresponding values of  $R_S$ . Fig. 2b (note that Fig. 2a is seen to give  $E = 1.8278$  at 3.950 GHZ which is virtually identical to the approximate analytical result above). The computed values of  $E$  inserted into (2) then gives the  $\lambda_0/\lambda_g$  predictions of Fig. 2c over 3.500 to 4.500 GHZ which also shows that for a WC-205. Examination of this figure shows that no significant difference exists in the guide wavelength of the subject CWPS between Case 1 and Case 2 (i.e., at a given frequency,  $\lambda_g$ , remains virtually constant during compression or expansion). Thus the degree-increment in phase-shift,  $D\Phi$ , through the CWPS due to a change in its length of DL can be written as:

$$D\Phi = 360(DL/\lambda_g) = 360(\lambda_0/\lambda_g)(DL/\lambda_0) \quad (7)$$

For example, at 3.950 GHZ (where  $\lambda_0/\lambda_g = 0.5288$ ),  $D\Phi = (63.668)DL$  degrees, where DL is in inches, a plot of which is given in Fig. 2(d).

### IV. MEASUREMENTS

The subject CWPS was connected to a conical horn antenna and placed in an anechoic chamber to receive the on-axis signal radiated from a source-horn in the far-field as shown

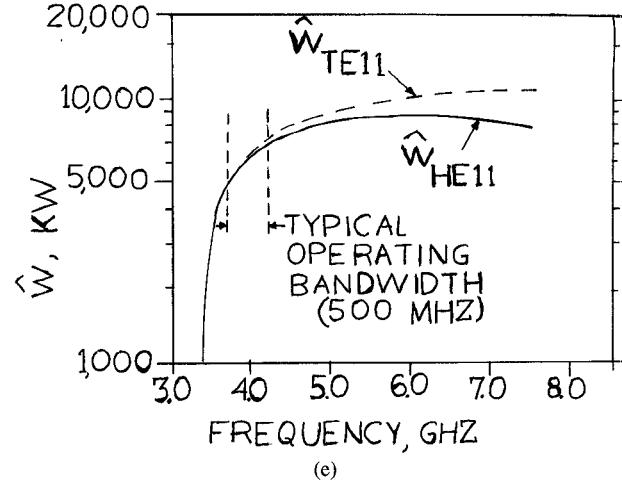


Fig. 2. *Continued.* (e) Predicted peak C.W. breakdown power of the  $HE_{11}$  mode in subject CWPS and of  $TE_{11}$  mode in WC-205.

in the photographs of Fig. 3. The other end of the CWPS was connected to a circular to rectangular waveguide transition having a coaxial connection output, to which was attached a flexible, but fixed-length, coaxial cable that was connected to a Model 1750 Scientific Atlanta receiver. The received-signal's phase was then measured as the CWPS was stretched. This was done in increments of 0.50 in. in  $L$  while holding the distance between receiving and transmitting horn apertures constant, as depicted on the insert of Fig. 2(d) that also shows the measured-results for ease of comparison to the predicted values. It is seen that the agreement between the two is satisfactory, though it could be improved by superior construction of the exterior rods supporting the CWPS to prevent the small transverse-bulging that was present (which was also found to cause a cross-polarized component of about -20 db, though the effect on the VSWR was trivial). Similarly, satisfactory results were obtained at other frequencies (but are not shown for brevity).

### V. APPLICATION TO AN ARRAY OF HPM DUAL-REFLECTOR ANTENNAS

If, now, the subject CWPS is connected to the horn of a dual-reflector antenna (DRA), the horn can be moved axially to control the phase of the emitted signal. It is seen from Fig. 2(d) that to realize a phase shift of  $360^\circ$  requires about a  $6''$  (i.e., about a  $2\lambda_0$ ) motion; similarly, a phase change of  $\pm 180^\circ$  would require about a  $\pm \lambda_0$  motion. In a typical DRA, such an axial horn motion introduces a defocusing gain drop that is acceptably low, as can be quickly estimated using the equivalent parabola concept [5]. Thus, since a typical DRA has an equivalent parabola with a focal length to a diameter ratio near unity use of well known gain-drop versus axial horn-motion curves [6], [7] gives about a 0.3-dB gain drop due to this  $\pm \lambda_0$  motion, which is tolerable.

The peak power capacity of a DRA is, primarily, limited by that of the input waveguide, or here, the CWPS. The expression of the C.W. power flow,  $W_{HE11}$ , of the  $HE_{11}$  mode in the CWPS can be obtained by using the transverse

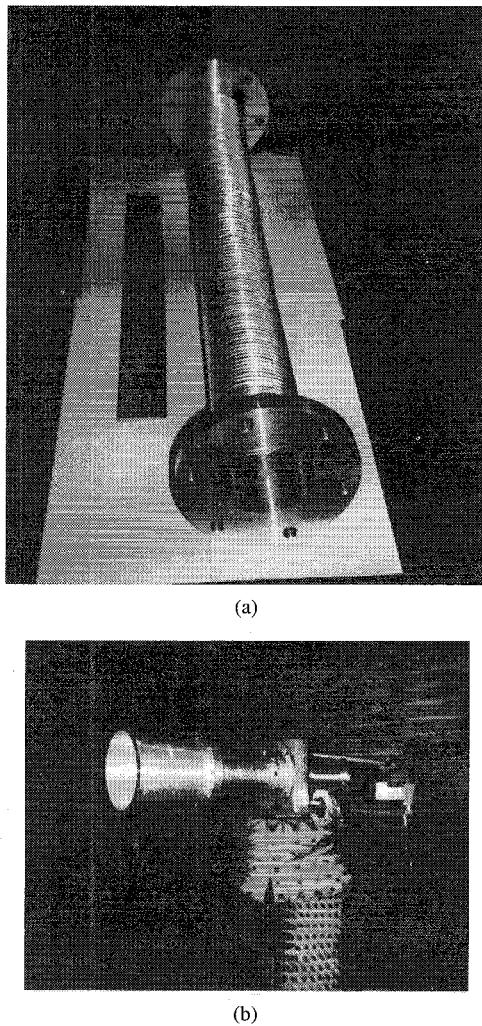


Fig. 3. Photographs of subject CWPS. (a) Per se. (b) View showing connection to conical horn for phase measurement.

field components of (1) and integrating Poynting's vector over the cross-section of diameter (Fig. 2(a)). This procedure is straightforward but tedious and gives (in watts):

$$W_{HE11} = (\pi a^2) |\epsilon_0|^2 W_{11} / Z_0 \quad (8)$$

where  $a$  is in cm and  $|\epsilon_0|$  in V/cm, with:

$$W_{11} = (AB - F) / D \quad (9)$$

where

$$\begin{aligned} A &= (E^2 / 4) [J_0^2(E) + J_1^2(E)] - J_1^2(E) / 2, \\ B &= (E^2 / S) [J_0(E) - J_1(E) / E]^2 + S J_1^2(E), \\ S &= [1 - (E/C)^2]^{1/2}, \\ F &= E J_1^3(E) [J_0(E) - J_1(E) / E] (S^{-1} - S) / 2, \\ D &= [E J_1(E)]^2. \end{aligned}$$

It is now noted that (since here  $R_S \approx 0$ ) that  $|\epsilon_0|/2$  is the peak magnitude of the  $E_\rho$  field on the guide's axis, as seen from (1). Thus, by setting  $|\epsilon_0|/2 \approx E_B = 30$  kV/cm (the ideal

breakdown field strength of air under STP) and denoting the corresponding value of  $W_{HE11}$  by  $\hat{W}_{HE11}$ , (10) gives:

$$\hat{W}_{HE11} / \lambda^2_0 = 760 C^2 W_{11} \quad (10)$$

as the peak power capacity (in kilowatts/cm<sup>2</sup>) of the CWPS before breakdown can occur. This expression was programmed and gives the results of Fig. 2(e) for the subject CWPS as well as a WC-205. The two curves are seen to be close over the typical operating bandwidth, over which peak-power capacities between 5 to 7 megawatts are predicted (usually this is reduced by a factor of 2 to 4 for conservative estimates). Now, because the fields of the subject  $HE_{11}$  mode are close to those of the  $TE_{11}$  mode (since  $E \approx E_0$ ,  $R_S \approx 0$ ), from (1) the  $E_\rho$  field at  $\rho = a$  is then about 4 dB ( $[2J_1(1.84)]/1.84 = 0.63$  times) lower than that at  $\rho = 0$ . However, it is known [8] that the electric fields at the corners of the corrugations can be increased by 2 to 3 times if the corners are round, and even more for square corners.

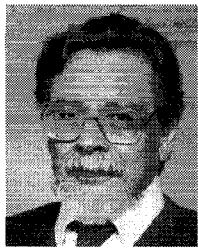
Hence, the  $E_\rho$  field at  $\rho = a$  may actually become greater than that at  $\rho = 0$  causing (10) to be an optimistic prediction, as is best determined by experiment.

## VI. CONCLUSION

The above theoretical and experimental results show that a thin-wall CWPS, having a diameter corresponding to a  $C = ka = 2.2$  and many shallow corrugations per free space wavelength, has, at a given frequency, a virtually constant phase velocity (or guide wavelength) independent of its maximum compression or expansion. As such, it can be used as a direct phase-shifter to give phase changes proportional to the change in its length, over a range of  $\pm 180^\circ$ , as governed by (7). Also, predictions disclose that it should be capable of carrying a peak C.W. power bounded by (10), as discussed. Thus, it is a candidate for a phase shifter application in an array of HPM dual-reflector antennas.

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